Oberseminar Wintersemester 2012/2013:

Drinfeld modules

Tuesday, 4 - 6 pm

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In this seminar we study the foundations of Drinfeld's theory of elliptic modules and elliptic sheaves. Elliptic modules are analogues in positive characteristic of elliptic curves in characteristic zero. In particular they have the same rich arithmetic theory as elliptic curves but are much more concrete algebraic objects than elliptic curves. One can therefore develop very similar arithmetic properties for elliptic modules using much less machinery.

Some topics we will discuss in the seminar are analytic uniformization for elliptic modules, morphisms of elliptic modules, isogeny classes of elliptic modules over finite fields (analog of Honda-Tate theory), Tate modules, moduli spaces of elliptic modules.

Afterwards we will translate elliptic modules into the framework of vector bundles, so called elliptic sheaves.

The moduli space of Drinfeld modules plays a crucial role for the Langlands correspondence for GL_r over function fields over finite fields. In fact in analogy to Eichler-Shimura theory for elliptic curves one decomposes the ℓ-adic étale cohomology of the moduli space by the action of Hecke operators and a Galois action which allows one to associate Galois modules to automorphic forms. At the end of the seminar we will sketch certain steps in this complicated construction initiated by Drinfeld. In fact using a generalization of elliptic modules, so called Drinfeld shtukas, Lafforgue has completed the proof of the global Langlands correspondence in positive characteristic [4].

The prerequisites for the seminar are a basic knowledge of algebraic geometry. Basic results from rigid analysis will be introduced during the seminar. It is motivationally helpful but not strictly necessary to be familiar with the arithmetic of elliptic curves.

Talks:

1: Overview (Moritz Kerz)

2: Additive Polynomials (NN)

Explain the elementary theory of additive polynomials [3], Sec. 1.1, 1.2, 1.6, 1.8.

3: Background on rigid analysis (NN)

Explain the content of [3] Chap. 2.

4: Lattices in $\mathbb{C}_\infty$ and elliptic modules I (NN)

We study lattices in $\mathbb{C}_\infty$ (the completion of the algebraic closure of $\mathbb{F}_p((t))$). Associated to a lattive one defines an entire function on $\mathbb{C}_\infty$ which has the lattice as a kernel. By transport of sturcture this gives rise to an elliptic module over $\mathbb{C}_\infty$. [1] Sec. 1.1; [3] Sec. 4.1-4.4.

5: Lattices in $\mathbb{C}_\infty$ and elliptic modules II (NN)
Define Rank and Height of an elliptic module and explain the rigid analytic uniformization theorem. [3] Sec. 4.5, 4.6.

6: Morphisms of elliptic modules (NN)
Morphisms of elliptic modules have very similar properties to morphisms of elliptic curves. Some basic facts are established in this talk. [3] Sec. 4.7, 4.9; [5] Sec. 2.1.

7: Analytic description of the moduli space (NN)
Explain how one can write the moduli space of elliptic modules as a homogeneous space over $\mathbb{C}_\infty$. Explain also the adelic description of this homogeneous space. [2] Sec. II.4,II.5.

8: Elliptic modules over finite fields and their isogeny classes (NN)
Recall first some background from the theory of central divison algebras over fields [3] 4.11, then present [3] Sec. 4.12; [5] Sec. 2.2, 2.3.

9: Elliptic modules over schemes and their moduli spaces (NN)
Define algebraic families of elliptic modules and their level structure. Show that their the moduli functor is representable by a smooth affine scheme. [5] Sec. 1.3-1.5.

10: Hecke correspondeces and fixed points (NN)
Introduce the convolution algebra and its Hecke corresponcence action on the moduli space of elliptic modules. Explain the Lefschetz number and sketch its relation to orbital integrals if time permits (no proofs) [5] 3.1 and Theorem 3.6.5.

11: Elliptic sheaves (NN)
Define elliptic sheaves and show how they are related to elliptic modules [1] Sec. 3.1, 3.2.

12: Shtukas I
TBA

13: Shtukas II
TBA

Literatur

[1] Blum, A.; Stuhler, U. Drinfeld modules and elliptic sheaves. Vector bundles on curves -new directions (Cetraro, 1995), 110–193, Lecture Notes in Math., 1649, Springer, Berlin, 1997.

[2] Deligne, P.; Husemoller, D. Survey of Drinfeld modules. Current trends in arithmetical algebraic geometry (Arcata, Calif., 1985), 25-91, Contemp. Math., 67, Amer. Math. Soc., Providence, RI, 1987.

[3] Goss, D. Basic structures of function field arithmetic. Ergebnisse der Mathematik und ihrer Grenzgebiete, 35. Springer-Verlag, Berlin, 1996.

[4] Lafforgue, L. Chtoucas de Drinfeld et correspondance de Langlands. Invent. Math. 147 (2002), no. 1, 1-241.

[5] Laumon, Gérard Cohomology of Drinfeld modular varieties. Part I. Geometry, counting of points and local harmonic analysis. Cambridge Studies in Advanced Mathematics, 41. Cambridge University Press, Cambridge, 1996.

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