Oberseminar Sommersemester 2012:

Linear Algebraic Groups

Tuesday, 16 - 18 pm
Prof. Dr. Moritz Kerz

In this seminar we study the basic theory of linear algebraic groups over algebraically closed fields with the aim of understanding the properties of the root system of a reductive group. The root system is a basic concept for many important applications to representation theory, automorphic forms etc.

We follow Borel's book on "Linear Algebraic Groups".

Talks:

1: Overview (Moritz Kerz)

2: Background on Algebraic Geometry (Anna Fluder)
Give an overview of the definitions and results we need from Sections AG.5-AG.10 and AG.16-AG.17. Skip rationality questions for the talk.

3: Basics of linear algebraic groups (Matthias Blank)
Sections 1 and 2.

4: Lie algebra and Jordan decomposition (Bernhard Dietel)
Sections 3 and 4.

5: Homogeneous spaces (Philipp Graf)
One aim is to construct quotients of linear groups by closed subgroups. Skip most proofs. Sections 5,6,7.

6: Tori (Timo Keller)
Rationality questions should only be mentioned quickly, diagonalizable groups, roots, quotients by tori.

7: Solvable groups I (Ivan Barrientos)
Centralizers and conjugacy classes of semi-simple elements, Lie-Kolchin Theorem. Sections 9 and part of 10.

8: Solvable groups II (Alexander Voitovich)
Structure theorem of connected solvable groups (Thm. 10.6), one-dimensional groups are isomorphic to $\mathbb{G}_a$ or $\mathbb{G}_m$, Definition of Borel subgroups and conjugacy of Borel subgroups (Thm. 11.1). Section 10 and beginning of Section 11.

9: Borel subgroups (Yingeng Zhao)
Borel subgroups, Cartan subgroups, maximal tori, theorem of Chevalley (Thm. 11.16), Weyl group, radical, reductive and semi-simple groups. Section 11.

10: Roots of a reductive group (NN)

Further properties of Cartan subgroup, regular and singular tori, groups of semi-simple rank one, roots of a reductive group (Thm. 13.18). Sections 12 and 13.

11: Abstract root systems (Timo Keller)

Explain abstract root systems and show how a reductive group (over an algebraically closed field) gives rise to a root system. Section 14.1-14.8

12: Bruhat decomposition, parabolic subgroups, Levi subgroups (Bernd Schober)

Decomposition of a semi-simple group into simple factors, Bruhat decomposition, parabolic subgroups, Levi subgroups. Section 14.9 - 14.26

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